Abstract: In the paper the main characteristics of filtration process of grey water (obtained from the laundry) on polypropylene filter of various size of pores (5 μm and 20 μm) has been presented. In was found well applicability of Ruth’s equation for filtration process of grey water in constant pressure. It means that in presented studies process is due to the first stage of formation of so called wet filtration cake. Parameters of Ruth’s equation were determined. On their flow resistance involved in the filtration process: resistant of the filter medium $R_f = 1.94 \cdot 10^{11} \text{ m}^{-1}$ (filter FCPS 5) and $R_f = 1.89 \cdot 10^{11} \text{ m}^{-1}$ (filter FCPS 20) and the average specific cake resistance $c_0$ for FCPS 5 $= 4.84 \cdot 10^{12} \text{ m}^{-2}$ and $c_0$ for FCPS 20 $= 4.35 \cdot 10^{13} \text{ m}^{-2}$ were calculated. Because both filters are made from polypropylene their resistances $R_f$ have similar values. However $c_0$ of both filters differs by an order of magnitude. Filtration times are equal to 42.3 min and 22.1 min respectively. Efficiency of both filters versus time of filtration were presented on the figures. It was proved higher life time of the FCPS 20 filter.

Keywords: grey water, constant pressure filtration, Ruth’s equation

Introduction

Water is the source of life on Earth, indispensable in the growth of organisms that can not live without it. Bathing, garden irrigation, flushing toilets, washing the vehicles or washing the dishes are a group of human activities, which are carried out every day with water. With an increasing number people on Earth the demand for water also increases. Freshwater resources on Earth are limited and already seriously affected. Countries, which, due to their geographical location previously faced a problems of water deficit at the present time are facing a serious problem of water lack. Also in Poland there is a water deficit. Today water resources per capita in our country are two times lower than in Western Europe. This situation obliges to the implementation of actions to reduce water consumption. In the times when humanitarian organizations build wells in poor countries that their people can drink clean water, it is incomprehensible to waste potable water, i.e. for toilet flushing.

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Therefore in many countries, a technology for domestic grey water recycling is being developed. Water generated during bathing or laundry called “gray water” can be treated on site and re-used for flushing toilets and for other purposes such as washing cars, washing driveways, watering lawns, etc. [1]. For high efficiency re-use of domestic gray water, special dual plumbing systems are necessary [2–5]. The benefits of using such systems are enormous. First of all they can give a big savings of potable water, which has a positive impact on the environment. Furthermore, use of dual plumbing system is a source of savings for households as it decreases consumption of tap water. Flushing toilets using gray water can conduct to 30 % savings. If the gray water recycling is connected also with rainwater recycling ie for irrigation of lawns, the savings can be even greater. In addition, the concept of using gray water for laundry may increase savings significantly.

Technology of gray water recycling is not very popular in Poland, but in the near future re-use of water may become necessary in every household. There is no doubt it brings enormous benefits therefore it should be promoted and regulated by law as soon as possible.

The aim of this study was to determine the basic characteristics of the gray water filtration on popular polypropylene filter cartridges. The filters with two different pore sizes were tested (5 and 20 microns). The flow resistance and overall filtration performance were determined. Wastewater from laundry of heavily soiled work clothes has been used.

**Theoretical principles of surface filtration**

Filtration is a process of solid-liquid separation [6] on porous filtration barrier by keeping the solids and passing the fluid. Liquid produced after filtration process is called filtrate. Filtration is the primary method of purifying water used by nature for millions of years. Layers of very clean groundwater were formed by rainwater filtration by various layers of soils. Filtration process in environmental engineering is used for water purification, urban and industrial wastewater treatment, as well as gray water treatment. These methods are based on the mechanisms that occur in the nature, however, they are conducted at much higher speed.

The use of filtration barriers with very small size pores (from 10–4 to 10–1 microns) can even separate solute from the solvent. This is done in the process of ultrafiltration or reverse osmosis. These processes, known as membrane processes are still very expensive.

Depending on the concentration of solids or the suspension in the filtrate we can distinguish cleansing filtration (solids concentration <0.1 %) and the separating filtration (solids concentration >1 %). During cleansing filtration the filtrate is obtained, during the separating filtration valuable sludge and also filtrate are obtained.

Special filtration barrier is the basic element of the filter. This could be the fabric filters, sand filters, porous masses, plastics or even its own sludge filter (so-called filter cake). The nature of filtration process depends on the type of filter cartridge:
– Volume filtration – filtration occurs through the piled layers of sand, slag, gravel, etc. Such filters are used for potable water and waste water clarification. In an event of discoloration of filtered medium, the diatomite, activated carbon or bleaching soil can be used;

– Surface filtration – a porous mass, plastics, fabric filters or filter cake is being used.

Filters used in the surface filtration begin to work properly as soon as the filter cake made from the corresponding layer of the sludge is being created. The thickness of the filter cake, which depends on the form of solids reduces the filtration velocity. When we are dealing with compressible sludge, filter cake is formed very quickly. Due to the low permeability of such cake, the filtration speed decreases rapidly. In such cases the filtration support like diatomite, bleaching soil, glass wool, wood chips or activated carbon is necessary.

**Filtration’s rate**

The ideal conditions for filtration [7] is assumed to be the laminar flow of liquid through a filter layer, which is made of a layer of inelastic and incompressible grains. Grains should have perfectly smooth and spherical surfaces. The filter barrier should be inelastic and incompressible, having perfectly formed, uniform cylindrical pores. For such conditions the Darcy’s equation is correct:

\[
W = \frac{dV}{d\Theta \cdot S} = \frac{\Delta P}{\mu (R_{0S} + R_f)}
\]

(1)

where: 

- \(W\) – filtration rate, per unit area of the filter \([m^3/m^2s]\);
- \(\Delta P\) – pressure difference \([N/m^2]\) (filtration pressure);
- \(V\) – volume of filtrate \([m^3]\);
- \(\Theta\) – time \([s]\);
- \(\mu\) – dynamic viscosity of liquid \([N \cdot s/m^2]\);
- \(R_{0S}\) – sludge resistance \([1/m]\);
- \(R\) – filter barrier resistance \([1/m]\);
- \(S\) – filtration surface area \([m^2]\).

The resistance of a filter cake layer (which is in fact a layer of sediment) is proportional to the thickness of cake \(L\) and resistivity \(\alpha\). Volume resistivity \(\alpha_0\) is a resistance of unit volume of sludge [7].

Assuming that the thickness \(L\) of the forming sludge is proportional to the volumetric concentration \(w_0\) of the solid in the filtered suspension we obtain the formula for the sludge resistance:

\[
R_{0S} = \alpha_0 \cdot L = \alpha_0 \cdot w_0 \cdot \frac{V}{S}
\]

(2)

where: 

- \(w_0 = \frac{V_0}{V}\);
- \(V_0\) – wet volume of sludge = \(S \cdot L\);
- \(V\) – volume of filtrate.
Substituting $R_{0S}$ into equation (1) we get the general equation of filtration:

$$\frac{dV}{d\Theta \cdot S} = \frac{\Delta P}{\mu \cdot \left(\alpha_0 \cdot w_0 \cdot \frac{V}{S} + R_f\right)}$$  \hspace{1cm} (3)$$

This equation relates the one-dimensional filtration ($S = \text{const.}$) where the sludge builds up in only one direction – on a flat surface. In real conditions, the filtration pressure is measured (usually equal to the pressure difference between the one produced by the pump and the atmospheric pressure). It is possible to distinguish the filtration under constant pressure or the filtration at constant rate. It is also possible to consider complex option of filtration when both parameters are variable.

**Filtration under constant pressure, $\Delta p = \text{const.}$**

After the integration of equation (3) in the range of 0 to $\Theta$, and from 0 to $V$ the filtration equation is as follows:

$$V^2 + 2V \cdot \frac{R_f \cdot S}{\alpha_0 \cdot w_0} = 2 \cdot \frac{\Delta P \cdot S^2}{\mu \cdot \alpha_0 \cdot w_0} \cdot \Theta$$  \hspace{1cm} (4)$$

where: $V$ – volume of filtrate [m$^3$];

$S$ – filtration surface area [m$^2$];

$\alpha_0$ – volume resistivity of incompressible sludge [1/m$^2$];

$w_0$ – the ratio of the sludge volume versus the filtrate volume [m$^3$/m$^3$];

$\Theta$ – filtration time [s].

According to the filtration conditions, the values of coefficients occurring at $V$ and $\Theta$ are constants. It is assumed that:

$$\frac{R_f \cdot S}{\alpha_0 \cdot w_0} = C \ [m^3] \quad \text{and} \quad \frac{2 \cdot \Delta P \cdot S^2}{\mu \cdot \alpha_0 \cdot w_0} = K \ [m^6/s]$$  \hspace{1cm} (5)$$

where: constants $C$ and $K$ describes the filtration processes through the barrier and the sludge; $C$ – filtration constant which characterizes hydraulic resistance of the filtration barrier. It means the volume of filtrate which should occur for sludge resistance equal to the one of the real filtration barrier, $K$ – filtration constant taking into account the conditions of the filtering process and physicochemical properties of sludge and liquid.

Substituting these values into equation (4) one can obtain the following equation:

$$V^2 + 2 \cdot V \cdot C = K \cdot \Theta$$  \hspace{1cm} (6)$$

This is the Ruth’s equation [8, 9] and the $C$ and $K$ are called Ruth’s coefficients. After differentiating equation (6) with respect to the filtration time, we obtain an equation describing the rate of filtration is obtained:
\[
\frac{dV}{d\Theta} = \frac{K}{2 \cdot (V + C)} \tag{7}
\]

The inverse of this equation is the following correlation:

\[
\frac{d\Theta}{dV} = \frac{2 \cdot V}{K} + \frac{2 \cdot C}{K} \tag{8}
\]

The illustration of this equation is a line in \(d\Theta/dV\) coordinate system (if the coordinates are not too far apart from each other, and if their values are not close to zero). The time needed to filtrate can be expressed as: \(t_p = C^2/K\).

Mechanical separation process consists of two consecutive phases:

– The phase of free filtration at constant pressure when the filter cake is formed,
– The phase of cake solidification under constant pressure. During this phase, the water bind with the previously formed cake is removed by the filter as a result of volume change caused by the mechanical stress.

Before the increasing filter cake reaches the plane of outlet, the filtration rate can be expressed by the equation (8). If the filtration cake will fill the space between the membrane and the plane of the outlet filter, the filtration surface is reduced and the above equation has to be modified [10, 11].

**Experimental**

**Research methodology and apparatus**

Analysis of gray water filtration was carried out at the appropriate test rig. The device consisting of Aquafilter® FHP-3 body and the appropriate filter cartridge has been used. There were two manometers on the input and output of the system. The diagram of the test rig is shown on Fig. 1.

![Test apparatus diagram](image)

**Fig. 1.** Test apparatus, the filter body with cartridge is installed on a tripod: A – body, B – head, C – retaining ring, D – vent, E – drain valve, F – valve, G – manometer
Filter consisted of transparent body, polypropylene mounting ring and the polypropylene head with brass 3/4 inch thread. The vent which was placed on the filter’s head was the additional element of the filter. It was used to remove the accumulated air from the interior of the cartridge. The body had a drain valve to remove contaminants from the filter’s cartridge. The Aquafilter® filter cartridges FCPS 5 (5 microns) and FCPS 20 (20 microns) were placed inside the filter’s body. Wastewater was pumped into the filter by the submersible pump with a lift height \( H_{\text{max}} = 5 \text{ m} \) and a pump efficiency \( Q_{\text{max}} = 6.5 \text{ m}^3/\text{h} \). Filters were mounted on a suitable stand. Filtration was carried out at a constant pressure.

**The source and basic quality parameters of grey water**

To carry out the experiments the gray water produced during laundry of heavily soiled clothing in domestic washing machine was used. Single input of clothing weighed about 3 kg. In all cases of grey water preparation the wash cycle included washing at 40 °C and three rinses. The color scheme of the laundry (dark colors) and the amount of the same detergent has been retained (150 cm³). Work cycle of the washing machine lasted about two hours. At this time the 50 liters of gray water has been obtained. The washing was performed on the same day as experiments to eliminate decaying of the grey water. The quality of 6 grey water samples of has been examined. Sample start gray water quality parameters were as follows: pH 9.53; suspension 707 mg/dm³; turbidity 996 NTU; COD 4480 mgO₂/dm³; BOD₅ 430 mgO₂/dm³; TOC 558 mgC/dm³. The experiments was carried out for two different filter cartridges: FCPS 20 microns and FCPS 5 microns.

**Results and discussion**

The purpose of this experiment was test the impact of filter clogging on the amount of obtained filtrate. The principles of the filtration theory and the Ruth’s equation were used for analysis of the experimental results.

The following experiments have been conducted:

– The filtrate volume at equal time intervals (every 4 min) has been determined;
– The time to obtain a constant volume of filtrate (2 litres) has been determined;
– The pressures on input and output of the filter have been observed.

**Filtration through FCPS 5 filter**

On the basis of experimental data the graph describing volume of filtrate versus filtration time has been prepared (Fig. 2). The curve fits well to Ruth’s equation (6), it shows that the filtration time is proportional to the square of the resulting filtrate’s volume. Flattening of the filtration curve during the time is caused by increasing filter cake and thus grows of the flow resistance. Consequently filter clogging occurs (which
can be observed on the graph) just after 25 minutes of the experiment. On the basis of
the Ruth’s curve it is possible to predict the efficiency of the filters and the need for
filter flushing or replacement.

![Graph showing filtrate volume V versus filtration time θ for filter FCPS 5.](image)

\[ V^2 + 2 \cdot V \cdot C = K \cdot θ \]
\[ V^2 + 2 \cdot V \cdot 37.8 = 33.8 \cdot θ \]

Fig. 2. The filtrate volume V versus the time of filtration (filter FCPS 5), ◆ – experimental points, the
theoretical curve according to equation (5)

Observing the gray water filtration through the FCPS 5 filter cartridge was found that
the filter blocked quickly.

In order to determine the Ruth’s K and C coefficients, the filtration curve (equation
(6)) shown in Fig. 3 was presented in the form of \( \frac{dQ}{dV} \) versus \( V \) (equation (8)).

![Graph showing dependence of inverse filtration rate \( \frac{dQ}{dV} \) on the volume of filtrate \( V \) for FCPS 5.](image)

Section 0–A corresponds to the \( 2C/K \) and the section |0–B| corresponds to \( |C| \) value.
They are equal to: \( |0,A| = 2.24 = 2C/K; |0,B| = |-37.8| = C \). On the basis of these data
coefficients: $K = 33.8 \text{ [dm}^6/\text{min}], \ C = 37.8 \text{ [dm}^3]\text{] and the filtration time } t_p = C^2/K \text{ [min]} = 42.3 \text{ min has been calculated.}

**Filtration through FCPS 20 filter**

Using the filter with bigger pores (FCPS 20) the duration of filtration process without blocking flow was explained. The time intervals between the measurements have not been changed the graphs showing the filtrate volume versus the filtration time and $dQ/dV$ versus $V$ have been prepared (Fig. 4 and 5).

![Graph](image)

*Fig. 4. The filtrate volume $V$ versus the time of filtration (filter FCPS 20), ◆ – experimental points, the theoretical curve according to equation (5)*

![Graph](image)

*Fig. 5. Dependence of inverse filtration rate $dQ/dV$ on the volume of filtrate $V$ (FCPS 20)*

Similar than before: $|0,A| = 2.18 = 2C/K; |0,B| = |–20.4| = C$ has been determined. On their basis determined: $K = 18.8 \text{ [dm}^6/\text{min}]; \ C = 20.4 \text{ [dm}^3]\text{] and the filtration time } t_p = C^2/K \text{ [min]} = 22.1 \text{ min.}
Filtration resistance

On the basis of calculated Ruth’s $K$ and $C$ constants the following filtration properties has been calculated: $\alpha_0$ – volume resistivity of the sludge and $R_f$ – filter barrier resistance. For the calculations the following data has been assumed:

- Filtrations pressure $\Delta p = 0.35$ bar $= 0.35 \cdot 10^5$ Pa;
- Surface area of filters $S = 0.0471$ m$^2$;
- Dynamic viscosity of grey water $\mu = 11.4 \cdot 10^{-4}$ Pa $\cdot$ s;
- The volumetric concentration of the solid $w_0$ estimated based on the thickness of the filter cake sludge: $w_0 = 0.05$ (for FCPS 5 filter) and $w_0 = 0.01$ (for FCPS 20 filter);
- Constant $K = 33.8$ dm$^6$/min (FCPS 5 filter);
- Constant $K = 18.8$ dm$^6$/min (FCPS 20 filter);
- Constant $C = 37.8$ dm$^3$ (FCPS 5 filter);
- Constant $C = 20.4$ dm$^3$ (FCPS 20 filter).

Volume resistivity of the sludge:

$$\alpha_0 = \frac{2 \cdot \Delta p \cdot S^2}{\mu \cdot K \cdot w_0}$$  \hspace{1cm} (9)

Filter barrier resistance:

$$R_f = \frac{C \cdot \alpha_0 \cdot w_0}{S} = \frac{2 \cdot \Delta p \cdot C}{\mu \cdot K}$$  \hspace{1cm} (10)

For FCPS 5 filter: $\alpha_0 = 4.84 \cdot 10^{12}$ m$^{-2}$; $R_f = 1.94 \cdot 10^{11}$ m$^{-1}$.
For FCPS 20 filter: $\alpha_0 = 4.35 \cdot 10^{13}$ m$^{-2}$; $R_f = 1.89 \cdot 10^{11}$ m$^{-1}$.

The calculations shows that the barrier resistance of the filter with smaller pores $R_f$ (FCPS 5) is slightly larger than than $R_f$ of filter with bigger pores (FCPS 20). Difference in the values of the barrier resistance $R_f$ is small, since both filters are made from the same hydrophobic material – polypropylene. However, volume resistivities of the sludge $\alpha_0$ differs by the order of magnitude.

Filter’s efficiency

The filter’s efficiency defined as the ratio of $[dV/d\Theta]_f$ to $[dV/d\Theta]_{f=0}$ versus filtration’s time is presented on Fig. 6 and 7.

The presented graphs show that the use of FCPS 20 filter is more beneficial than the FCPS 5 filter, due to the fact that for longer time filtration process it retains good efficiency.
Conclusions

The applicability of the Ruth’s equation for gray water filtration process (in undertaken scope of research) indicates on the first phase of the process: forming the wet filter cake [10].

The filter with smaller pores – FCPS 5 shows both a higher resistance of filter barrier (constant $C$) and higher filter constant $K$, related to the sludge properties, than the filter with larger pores FCPS 20.

Values of $C$ constants equals 33.8 [dm$^3$] for FCPS 5 filter and 18.8 [dm$^3$] for FCPS 20 filter but the values of $K$ constants equals 37.8 [dm$^6$/min] and 20.4 [dm$^6$/min] respectively.

According to the $K$ and $C$ constants filtration’s time $t_p = C^2/K$ has been calculated. Its values equals 42.3 min for FCPS 5 filter and 22.1 min for FCPS 20 filter.
On the basis of gray water properties, filter properties and K and C constants, using the equations 8 and 9 the sludge resistivity and resistance of the filtration barrier were determined.

For the FCPS5 filter these values equals: $\alpha_0 = 4.84 \cdot 10^{12} \text{ m}^{-2}$; $R_f = 1.94 \cdot 10^{11} \text{ m}^{-1}$, for the FCPS 20 filter: $\alpha_0 = 4.35 \cdot 10^{13} \text{ m}^{-2}$; $R_f = 1.89 \cdot 10^{11} \text{ m}^{-1}$.

The measured efficiency of the filters also confirms better parameters of FCPS 20 filter (Figs. 6 and 7).

In order to fully compare the properties of both filters, the physico-chemical properties of the filtrate has been analyzed which is discussed in a separate publication [12].

On a basis of literature review carried out in the work [11, 13] was found that the physical processes conducted alone are not sufficient to ensure proper reduction of organic compounds and chemicals (including surfactants) contained in the gray water.

The combination of biological, chemical and physical filtration processes is the best method for the proper gray water recycling [14–19].

References

CHARAKTERYSTYKA FILTRACJI WODY SZAREJ NA FILTRACH POLIPROPYLENOWYCH

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Abstrakt: W pracy przedstawiono podstawową charakterystykę procesu filtracji wody szarej (otrzymanej z prania) na polipropylenowych wkładach filtracyjnych o różnym mikronaślu (5 μm, 20 μm). Stwierdzono stosowność równania Rutha do procesu filtracji wody szarej, co świadczy o tym, że w podjętych badaniach przebieg procesu dotykał pierwszej fazy – tzw. formowania mokrego placka filtracyjnego. Wyznaczono parametry równania Rutha oraz na ich podstawie obliczono wartości oporów przegród filtracyjnych oraz objętościowych oporów właściwych osadu. Filtr o mniejszym mikronaślu (FCPS 5) wykazywał nieco większy opór przegrody filtracyjnej \( R_f = 1,94 \cdot 10^{11} \text{ m}^{-1} \) niż filtr o mikronaślu wyższym FCPS 20 \( R_f = 1,89 \cdot 10^{11} \text{ m}^{-1} \). Niewielkie różnice w wartościach oporów wynikają z charakterystyki obydwu filtrów. Obydwa zbudowane są ze spienionego polipropylenu. Natomiast wartości obliczonych objętościowych oporów właściwych osadu różniły się o rzęd wielkości \( c_0 \) dla filtra FCPS 5: 4,84 \( \cdot \) \( 10^{12} \text{ m}^{-2} \) i \( c_0 \) dla FCPS 20: 4,35 \( \cdot \) \( 10^{13} \text{ m}^{-2} \). Czasy filtracji wynosiły odpowiednio 42,3 min oraz 22,1 min. Oceniono również wydajność filtracji na obydwu wkładach filtracyjnych i przedstawiono je na wykresach w funkcji czasu, z których wynika dłuższa żywotność filtra FCPS 20.

Słowa kluczowe: szara woda, filtracja pod stałym ciśnieniem, równanie Rutha